Computing the Combined Effect of Measurement Errors and Non-Response using Factor Chain-type Class of Estimator

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In this article, we have suggested an efficient factor chain-type class of estimators in the presence of measurement error and non-response simultaneously. It is shown that several estimators can be generated from our proposed class of estimators. Mean Square Error of the proposed class of estimator are derived and compared with other existing estimators. The conditions under which proposed estimator is more efficient are obtained. A theoretical and empirical study has done to demonstrate the efficiency of this estimator over other existing estimators.

Keywords: Auxiliary variable, bias, mean square error, measurement errors, nonresponse, study variable

1. Introduction

In order to estimate the finite population mean under non-response conditions, Hansen and Hurwitz (1946) has suggested the method to estimate nonresponse by taking a subsample of nonrespondent group with the help of extra efforts and an estimator was developed by combining the information available from the response and non-response groups. Following the work of Hansen and Hurwitz (1946) based on the sample mean per unit, several authors have used this method in the conventional and alternative ratio and regression type estimation methods.

Authors including Cochran (1977), Rao (1986, 1990), Okafor and Lee (2000), Tabasum and Khan (2004), Sodipo and Obisesan (2007), Singh and Kumar (2008), Shabbir and Khan (2013) have studied the effect of non-response on different estimators of population parameters (mean, variance, etc.).

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In survey sampling there present another type of nonsampling error which is measurement errors while collecting information from individuals. Basically, measurement error is defined as the discrepancy between the recorded value provided by the respondent and the true value of a variable in the study. Estimating the population parameters using auxiliary information, many authors have addressed the problem in the presence of measurement errors. Fuller (1995) has discussed the impacts of measurement errors in linear and nonlinear regression modelling in their books on measurement errors. Shalabh (1997), Srivastva and Shalabh (2001), Maneesha and Singh (2002) studied the impact of measurement errors in ratio and regression method of estimation. Singh and Vishwakarma (2019) studied joint effect of measurement error and non-response. Also in the estimation of population mean, Singh and Shukla (1987) presented a family of the factor-type estimator and which results into ratio, product and dual-to-ratio and usual sample mean estimator for specific values of constant. Following above literature using auxiliary information an efficient factor chain-type class of estimator of population mean is proposed with the combined effect of measurement errors and non-response in sample survey. Since the proposed class of estimator is a factor chain-type class of estimators, several existing estimators viz. chain-ratio, Ratio to Product estimator, Dual to Ratio estimator etc. can be a member of this proposed class.

Suppose a random sample of size n be drawn from a finite population $U=(U_1, U_2, \ldots, U_N)$ of size N by using simple random sampling without replacement (SRSWOR) scheme. It is assumed that the population of size N can be divided into two non-overlapping strata of size N_1 and N_2. Stratum N_1 responding units out of N would respond on the first call and Stratum N_2 ($N_2 = N_1 - N$) non-responding units out of N would not respond on the first call but would respond on the second call. Following Hansen and Hurwitz (1946) technique, each sampled unit belongs to one of the three mutually exclusive groups:

1. the n_1 response units supply information to the mail questionnaire,
2. the $(n_2 - r)$ units who did not supply information to the mail questionnaire and were not contacted again by the personal interview method, and
3. the r ($r = n_2/l$, $l > 1$) units is selected at random without replacement from the n_2 nonrespondent units who did not respond to the mail questionnaire but enumerated by personal interview, where l is the inverse sampling ratio.

It is assumed that (x_i^*, z_i^*) are recorded values instead of true values (X_i^*, Z_i^*) for ith (i = 1, 2,…, n) sampling units of two characteristics (x, z). Suppose the measurement errors can be express as $z_i^* = U_i^* + Z_i^*$, $x_i^* = V_i^* + X_i^*$ and are normally distributed with mean zero and variances S_u^2 and S_v^2 respectively. Also

$f_1 = \left(\frac{1}{n} - \frac{1}{N}\right)$, $\theta = \frac{W_2(l-1)}{n}$ and $W_2 = \frac{N_2}{N}$. Let $S_z^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(z_i - \bar{Z}\right)^2$ and $S_x^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2$ be the population variances of the study variable z and auxiliary variable y respectively. Let $S_{z(2)}^2 = \frac{1}{(N_2-1)}\sum_{i=1}^{N_2}\left(z_i - \bar{Z}\right)^2$ and $S_{x(2)}^2 = \frac{1}{(N_2-1)}\sum_{i=1}^{N_2}\left(x_i - \bar{X}\right)^2$ be the variances of the variables z and x respectively for the nonresponding part of population. Let $S_u^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(u_i - \bar{U}\right)^2$ and $S_v^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(v_i - \bar{V}\right)^2$ be the error variance associated with study variables z and auxiliary variable x respectively and $S_{u(2)}^2 = \frac{1}{(N_2-1)}\sum_{i=1}^{N_2}\left(u_i - \bar{U}\right)^2$ and $S_{v(2)}^2 = \frac{1}{(N_2-1)}\sum_{i=1}^{N_2}\left(v_i - \bar{V}\right)^2$ be the error variance of variables z and x respectively for the nonresponding part of population. Let $\bar{z} = \frac{1}{n}\sum_{i=1}^{n} z_i$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ be the unbiased estimator of population mean $\bar{Z}$ and $\bar{X}$ of variable z and x respectively. Let $R = \frac{\bar{Z}}{\bar{X}}$ be the ratio of population means of study and auxiliary variable.

2. Proposed Class Estimator

Influenced by existing theory on the non-response and measurement error we proposed an efficient chain-factor type class of estimator of population mean $\bar{Z}$ in the presence of non-response and measurement error simultaneously as

$$T_k = \bar{z}^* m_1(1,0)\left[\frac{\phi\{\eta_1(k)\}}{\phi\{\eta_2(k)\}}\right] \tag{2.1}$$

where, $\phi\{\eta_i(k)\} = \eta_i(k) + \{1 - \eta_i(k)\} m_2(0,1)$; i=1, 2.

$$\eta_1(k) = \frac{f\,B}{A + f\,B + C},\quad \eta_2(k) = \frac{C}{A + f\,B + C}$$

$A = (k - 1)(k - 2)$, $B = (k - 1)(k - 4)$, $C = (k - 2)(k - 3)(k - 4)$, $f = \frac{n}{N}$

and k is suitably chosen constant to minimize the mean square error of the proposed estimator.

$$m_1(a,b) = \left(\frac{\bar{X}}{\bar{x}^*}\right)^a \left(\frac{\bar{X}}{\bar{x}'^*}\right)^b \text{ and } m_2(a,b) = \left(\frac{\bar{X}}{\bar{x}'^*}\right)^a \left(\frac{\bar{X}}{\bar{x}}\right)^b$$

where $\bar{x}' = \dfrac{N\bar{X} - n\bar{x}}{(N - n)}$ and $\bar{x}'^* = \dfrac{N\bar{X} - n\bar{x}^*}{(N - n)}$

Since the estimator T_k is a class of estimator and this is biased estimator of the population mean $\bar{Z}$. Therefore, the resulting members of estimators are also biased estimators of population mean $\bar{Z}$. In order to obtain the bias $B(T_k)$ and mean square error $MSE(T_k)$, we introduce some following transformations:

$$\psi_z^* = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \left(z_i^* - \bar{Z}\right) \tag{2.2}$$

$$\psi_U^* = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} U_i^* \tag{2.3}$$

$$\psi_x^* = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \left(x_i^* - \bar{X}\right) \tag{2.4}$$

and

$$\psi_V^* = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} V_i^* \tag{2.5}$$

Adding (2.2) and (2.3), we have $\psi_z^* + \psi_U^* = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \left(z_i^* - \bar{Z}\right) + \frac{1}{\sqrt{n}} \sum_{i=1}^{n} U_i^*$

Multiplying both sides by $\frac{1}{\sqrt{n}}$, we have

$$\frac{1}{\sqrt{n}}\left(\psi_z^* + \psi_U^*\right) = \frac{1}{n} \sum_{i=1}^{n} \left(z_i^* - \bar{Z}\right) + \frac{1}{n} \sum_{i=1}^{n} U_i^* \tag{2.6}$$

On simplification and using $n = n_1 + (n_2 - r) + r$, we get

$$\bar{z}^* = \bar{Z} + \frac{1}{\sqrt{n}}\left(\psi_z^* + \psi_U^*\right) \tag{2.7}$$

Similarly, from (2.4) and (2.5), we get

$$\bar{x}^* = \bar{X} + \frac{1}{\sqrt{n}}\left(\psi_x^* + \psi_V^*\right) \tag{2.8}$$

Further, we have the following expectations.

$$E\left(\frac{\psi_z^* + \psi_U^*}{\sqrt{n}}\right)^2 = f_1\left(S_z^2 + S_u^2\right) + \theta\left(S_{z(2)}^2 + S_{u(2)}^2\right)$$

$$E\left(\frac{\psi_x^* + \psi_V^*}{\sqrt{n}}\right)^2 = f_1\left(S_x^2 + S_u^2\right) + \theta\left(S_{x(2)}^2 + S_{v(2)}^2\right)$$

$$E\left\{\left(\frac{\psi_z^* + \psi_U^*}{\sqrt{n}}\right)\left(\frac{\psi_x^* + \psi_V^*}{\sqrt{n}}\right)\right\} = f_1 \rho_{zx} S_z S_x + \theta \rho_{zx(2)} S_{z(2)} S_{x(2)}$$

Following above transformation, the estimator T_k can be written as

$$T_k = \left(\bar{Z} + W_Z\right)\frac{\bar{X}}{\left(\bar{X} + W_x^*\right)}\left[\frac{\eta_1(k) + \{1 - \eta_1(k)\}\dfrac{\bar{X}}{\left(\bar{X} + W_x\right)}}{\eta_2(k) + \{1 - \eta_2(k)\}\dfrac{\bar{X}}{\left(\bar{X} + W_x\right)}}\right] \tag{2.9}$$

where $W_z = \left(\dfrac{\psi_z^* + \psi_U^*}{\sqrt{n}}\right)$ and $W_x^* = \left(\dfrac{\psi_x^* + \psi_V^*}{\sqrt{n}}\right)$

$$T_k = \left(\bar{Z} + W_Z\right)\left(1 + \frac{W_x^*}{\bar{X}}\right)^{-1}\left[\frac{\eta_1(k) + \{1 - \eta_1(k)\}\left(1 + \dfrac{W_x}{\bar{X}}\right)^{-1}}{\eta_2(k) + \{1 - \eta_2(k)\}\left(1 + \dfrac{W_x}{\bar{X}}\right)^{-1}}\right] \tag{2.10}$$

Thus, expressing T_k in terms of W_z, W_x^*, and W_x also neglecting the terms of W_z, W_x^*, and W_x having power greater than two we get

$$T_k - \bar{Z} = \left[W_Z - \frac{W_x^* \bar{Z}}{\bar{X}} + \eta(k)\frac{W_x \bar{Z}}{\bar{X}} + \frac{W_x^{*2}\bar{Z}}{\bar{X}^2} - \frac{W_x^* W_z}{\bar{X}} - \eta_2(k)\eta(k)\frac{W_x^2 \bar{Z}}{\bar{X}^2} \right. \tag{2.11}$$
$$\left. +\eta(k)\frac{W_x W_Z}{\bar{X}} - \eta(k)\frac{W_x W_x^* \bar{Z}}{\bar{X}} \right]$$

where, $\eta(k) = \eta_1(k) - \eta_2(k)$.

Taking expectations on both sides of the equation (2.11), we obtain the expression for the bias $B(T_k)$ is given as

$$B(T_k) = \left[f_1 \frac{\bar{Z}}{\bar{X}^2}\{1 - \eta(k) - \eta(k)\eta_2(k)\}\left(S_x^2 + S_v^2\right) + \theta\frac{\bar{Z}}{\bar{X}^2}\left(S_{x(2)}^2 + S_{v(2)}^2\right) \right. \tag{2.12}$$
$$\left. + f_1 \frac{1}{\bar{X}}\rho_{zx} S_z S_x \{\eta(k) - 1\} - \theta\frac{1}{\bar{X}}\rho_{zx(2)} S_{z(2)} S_{x(2)} \right]$$

Squaring and taking expectations on both sides of the equation (2.11), we obtain the expression for the mean square error $MSE(T_k)$ up to the first order of approximation as

$$MSE(T_k) = MSE(T_{1k}) + \mathrm{MSE}(T_{2k}) \tag{2.13}$$

where

$$MSE(T_{1k}) = \bar{Z}^2\left[f_1\left\{ C_z^2 + C_x^2\left(1 - \eta(k)\right)^2 - 2\rho_{zx} C_z C_x \left(1 - \eta(k)\right) \right\} \right. \tag{2.14}$$
$$\left. + \theta\left\{ C_{z(2)}^2 + C_{x(2)}^2 - 2\rho_{zx(2)} C_{z(2)} C_{x(2)} \right\} \right]$$

is the MSE of T_k without measurement error, and

$$MSE(T_{2k}) = \bar{Z}^2\left[f_1\left\{ \frac{S_u^2}{\bar{Z}^2} + \frac{S_v^2}{\bar{X}^2}\left(1 - \eta(k)\right)^2 \right\} + \theta\left\{ \frac{S_{u(2)}^2}{\bar{Z}^2} + \frac{S_{v(2)}^2}{\bar{X}^2} \right\} \right] \tag{2.15}$$

is the contribution of measurement error in the MSE of T_k.

3. Optimum Mean Square Error of Proposed class Estimator T_k

Since $\eta(k)$ is a function of k. Therefore, differentiating mean square error $MSE\,(T_k)$ with respect to $\eta(k)$ and equating to zero. The optimum value of $\eta(k)$ is obtained as

$$\frac{\partial}{\partial k}\eta^2(k) = 2\eta'(k)\eta(k) \text{ and } \eta'(k) \neq 0$$

$$\eta(k)_{opt} = \left(1 - \frac{\rho_{zx} C_z C_x}{C_x^2 + S_v^2 / \bar{X}^2}\right) \tag{3.1}$$

Now, by substituting the value of $\eta(k)_{opt}$ in equation (2.14) and (2.15), the minimum mean square error of the estimator T_k is obtained as

$$MSE(T_k)_{\min} = \bar{Z}^2\left[f_1 C_z^2 + \theta\left\{C_{z(2)}^2 + C_{x(2)}^2 - 2\rho_{zx(2)} C_{z(2)} C_{x(2)}\right\} + f_1\frac{S_u^2}{\bar{Z}^2} + \theta\left\{\frac{S_{u(2)}^2}{\bar{Z}^2} + \frac{S_{v(2)}^2}{\bar{X}^2}\right\} - f_1\left\{\rho_{zx}^2 C_z^2 \Big/ \left(1 + \frac{S_v^2}{S_x^2}\right)\right\}\right] \tag{3.2}$$

We can identify some of the members of the class of estimators T_k as

i.) $\bar{z}_{CR}^* = \bar{z}^* m_3(1,0) = \bar{z}^* \dfrac{\bar{X}}{\bar{x}^*}$ (3.3)

ii.) $\bar{z}_{CP}^* = \bar{z}^* m_3(-1,0) = \bar{z}^* \dfrac{\bar{x}^*}{\bar{X}}$ (3.4)

iii.) $\bar{z}_{RR}^* = \bar{z}^* m_2(0,1) = \bar{z}^* \dfrac{\bar{X}}{\bar{x}}$ (3.5)

iv.) $\bar{z}_{RP}^* = \bar{z}^* m_2(0,-1) = \bar{z}^* \dfrac{\bar{x}}{\bar{X}}$ (3.6)

v.) $\bar{z}_{SKR}^* = \bar{z}^* m_3(1,0) m_2(0,1) = \bar{z}^* \left(\dfrac{\bar{X}}{\bar{x}^*}\right)\left(\dfrac{\bar{X}}{\bar{x}}\right)$ (3.7)

vi.) $\bar{z}_{SKP}^* = \bar{z}^* m_3(-1,0) m_2(0,-1) = \bar{z}^* \left(\dfrac{\bar{x}^*}{\bar{X}}\right)\left(\dfrac{\bar{x}}{\bar{X}}\right)$ (3.8)

It can be seen that the estimators (i)–(vi) are well-known ratio and product-type estimators in sampling scheme. Then standard ratio and product estimators (i) and (ii) respectively were first proposed and studied by Cochran (1977). The usual ratio and product-type estimators (iii) and (iv) respectively first defined by Rao (1986) and the chain-type-ratio and product estimators (v) and (vi) respectively were proposed by Singh and Kumar (2008) in the presence of non-response.

Remark 1. Also, we observed that the class of estimator T_k reduce to the following form of estimators for the different value of k as shown in Table 1.

Table 1. The estimators fall under the proposed class of estimators for different choice of *k*

k	T_k	
	Estimators	**Proposed by**
1	$T_1 = \bar{z}^* \left(\frac{\bar{X}}{\bar{x}^*}\right)\left(\frac{\bar{X}}{\bar{x}}\right)$	Singh and Kumar (2008), Chain-ratio-type estimator
2	$T_2 = \bar{z}^* \left(\frac{\bar{X}}{\bar{x}^*}\right)\left(\frac{\bar{x}}{\bar{X}}\right)$	Ratio to Product-type estimator
3	$T_3 = \bar{z}^* \frac{\bar{x}'}{\bar{x}^*}$	Dual to Ratio-type estimator
4	$T_4 = \bar{z}^* \frac{\bar{X}}{\bar{x}^*}$	Cochran (1977), Ratio-type estimator

$$V\left(\bar{z}^*\right) = f_1 S_z^2 + \theta S_{z(2)}^2 + f_1 S_u^2 + \theta S_{u(2)}^2 \tag{3.9}$$

$$\begin{aligned} MSE\left(\bar{z}_{CR}^*\right) = {} & f_1\left[S_z^2 + R^2 S_x^2 - 2R\rho_{zx} S_z S_x\right] + \theta\left[S_{z(2)}^2 + R^2 S_{x(2)}^2 - 2R\rho_{zx(2)} S_{z(2)} S_{x(2)}\right] \\ & + f_1\left[S_u^2 + R^2 S_v^2\right] + \theta\left[S_{u(2)}^2 + R^2 S_{v(2)}^2\right] \end{aligned} \tag{3.10}$$

$$MSE\left(\bar{z}_{RR}^*\right) = f_1\left[S_z^2 + R^2 S_x^2 - 2R\rho_{zx} S_z S_x\right] + \theta S_{z(2)}^2 + f_1\left[S_u^2 + R^2 S_v^2\right] + \theta S_{u(2)}^2 \tag{3.11}$$

$$\begin{aligned} MSE\left(\bar{z}_{SKR}^*\right) = {} & f_1\left[S_z^2 + 4R^2 S_x^2 - 2R\rho_{zx} S_z S_x\right] + \theta\left[S_{z(2)}^2 + R^2 S_{x(2)}^2 - 2R\rho_{zx(2)} S_{z(2)} S_{x(2)}\right] \\ & + f_1\left[S_u^2 + 4R^2 S_v^2\right] + \theta\left[S_{u(2)}^2 + R^2 S_{v(2)}^2\right] \end{aligned} \tag{3.9}$$

4. Efficiency Comparison

In this section the proposed class of estimator T_k has been compared with respect to existing estimators $\bar{z}^*$, $\bar{z}^*_{CR}$, $\bar{z}^*_{RR}$, $\bar{z}^*_{SKR}$ respectively and result shown below:

(i). $MSE(T_k)_{min}$ will be more efficient than $V(\bar{z}^*)$ if MSE $(T_k)_{min} \leq V(\bar{z}^*)$, which provide

$$f_1 \geq \frac{\left(C_x^2+\frac{S_v^2}{\bar{X}^2}\right)}{\rho_{zx}^2 C_z^2 C_x^2}\left[C_{x(2)}^2+\frac{S_{v(2)}^2}{\bar{X}^2}-2\rho_{zx(2)}C_{z(2)}C_{x(2)}\right] \tag{4.1}$$

(ii). $MSE(T_k)_{min}$ is more precise than $MSE(\bar{z}^*_{CR})$, when MSE $(T_k)_{min} \leq MSE(\bar{z}^*_{CR})$

$$\rho_{zx} \leq \left[\frac{C_x}{C_z}+\frac{S_v^2}{C_z C_x \bar{X}^2}\right] \tag{4.2}$$

(iii). $MSE(T_k)_{min}$ is preferable over $MSE(\bar{z}^*_{RR})$ if $MSE(T_k)_{min} \leq MSE(\bar{z}^*_{RR})$, which give

$$f_1 \geq \frac{C_{x(2)}^2+\frac{S_{v(2)}^2}{\bar{X}^2}-2\rho_{zx(2)}C_{z(2)}C_{x(2)}}{\left(C_x^2+\frac{S_v^2}{\bar{X}^2}\right)-\rho_{zx}C_zC_x\left(2-\frac{\rho_{zx}C_zC_x}{C_x^2+\frac{S_v^2}{\bar{X}^2}}\right)} \tag{4.3}$$

(iv). $MSE(T_k)_{min}$ will dominate $MSE(\bar{y}^*_{SKR})$ if $MSE(T_k)_{min} \leq MSE(\bar{y}^*_{SKR})$, subsequently we get

$$\rho_{zx} \geq 2\left[\frac{C_x}{C_z}+\frac{S_v^2}{C_z C_x \bar{X}^2}\right] \tag{4.4}$$

5. Numerical Illustration

To validate the performance of the proposed estimator T_k under nonresponse and measurement error we have used secondary data sets from the Azeem and Haneef (2017) generated three populations of size N=5000 from the normal distribution with different choices of parameters by using R studio. We have divided the population into two non-overlapping strata N_1 and N_2. The parameters of the populations are as follows:

Population I.

N = 5000, $\overline{Z} = 4.927$, $\overline{X} = 4.924$, $S_z^2 = 102.007$, $S_x^2 = 101.412$, $\rho_{zx} = 0.995$, $S_u^2 = 8.862$, $S_v^2 = 9.001$,

N_1	N_2	$S^2_{z(2)}$	$S^2_{x(2)}$	$S^2_{u(2)}$	$S^2_{v(2)}$	$\rho_{zx(2)}$	W_2
4500	500	99.99174	99.87471	9.150544	8.756592	0.994916	0.10
4250	750	100.9428	100.8224	9.053862	8.766538	0.995535	0.15
4000	1000	104.2711	103.2349	8.821278	8.339179	0.995472	0.20

Population II.

N = 5000, $\overline{Z} = 4.927$, $\overline{X} = 5.0135$, $S_z^2 = 97.1206$, $S_x^2 = 95.958$, $\rho_{zx} = 0.995$,

$S_u^2 = 23.960$, $S_v^2 = 24.193$,

N_1	N_2	$S^2_{z(2)}$	$S^2_{x(2)}$	$S^2_{u(2)}$	$S^2_{v(2)}$	$\rho_{zx(2)}$	W_2
4500	500	97.02783	94.54578	22.80557	25.43263	0.994546	0.10
4250	750	98.27616	97.42674	23.27837	24.13829	0.994992	0.15
4000	1000	96.09359	94.71923	24.42978	23.03076	0.99467	0.20

Population III.

N = 5000, $\overline{Z} = 1.961$, $\overline{X} = 1.943$, $S_z^2 = 25.441$, $S_x^2 = 24.504$, $\rho_{zx} = 0.981$,

$S_u^2 = 6.040$, $S_v^2 = 6.224$,

N_1	N_2	$S^2_{z(2)}$	$S^2_{x(2)}$	$S^2_{u(2)}$	$S^2_{v(2)}$	$\rho_{zx(2)}$	W_2
4500	500	24.52749	23.61208	6.335415	5.589493	0.979112	0.10
4250	750	28.59666	27.55397	6.124221	6.299637	0.982175	0.15
4000	1000	25.87711	25.2131	5.938303	6.272239	0.982566	0.20

We have computed the percentage relative efficiencies (PRE) of the proposed estimator T_k and other existing estimators $\bar{z}^*_{CR}$, $\bar{z}^*_{RR}$, and $\bar{z}^*_{SKR}$, with respect to usual unbiased Hansen and Hurwitz (1946) estimator $\bar{z}^*$ through

$$\text{PRE} = \frac{V\left(\bar{z}^*\right)}{M\left(\tau\right)} \times 100 \tag{5.1}$$

where $\tau = \bar{z}^*$, $\bar{z}^*_{CR}$, $\bar{z}^*_{RR}$, $\bar{z}^*_{SKR}$, T_k and presented in Table 2 to 4.

Table 2. PRE of the estimators with respect to the Hansen and Hurwitz (1946) estimator $\bar{z}^*$ for population

N_1	N_2	l	$\bar{z}^*$	$\bar{z}^*_{CR}$	$\bar{z}^*_{RR}$	$\bar{z}^*_{SKR}$	T_k
4500	500	2	100.00	554.37	385.61	147.70	579.33
		4	100.00	544.97	262.24	167.98	575.01
		6	100.00	538.42	213.30	186.32	571.95
		8	100.00	533.60	187.04	203.00	569.67
4250	750	2	100.00	556.22	339.09	153.15	578.93
		4	100.00	550.21	221.84	182.42	574.58
		6	100.00	546.45	181.75	207.75	571.84
		8	100.00	543.87	161.51	229.89	569.96
4000	1000	2	100.00	576.63	302.89	159.25	583.95
		4	100.00	580.68	195.73	198.07	586.02
		6	100.00	582.85	162.64	230.36	587.21
		8	100.00	586.02	146.55	257.64	587.99

Table 3. PRE of the estimators with respect to the Hansen and Hurwitz (1946) estimator $\bar{z}^*$ for population II

N_1	N_2	l	$\bar{z}^*$	$\bar{z}^*_{CR}$	$\bar{z}^*_{RR}$	$\bar{z}^*_{SKR}$	T_k
4500	500	2	100.00	246.95	215.80	60.82	270.21
		4	100.00	246.43	181.12	69.44	265.57
		6	100.00	246.07	162.42	77.29	262.35
		8	100.00	245.79	150.73	84.47	259.98
4250	750	2	100.00	248.80	204.19	63.22	270.10
		4	100.00	250.83	165.72	75.88	265.86
		6	100.00	252.14	148.00	86.96	263.22
		8	100.00	253.05	137.80	96.72	261.43
4000	1000	2	100.00	246.89	204.45	63.14	269.75
		4	100.00	246.36	166.03	75.66	265.05
		6	100.00	246.02	148.27	86.61	262.13
		8	100.00	245.78	138.04	96.26	260.15

Table 4. PRE of the estimators with respect to the Hansen and Hurwitz (1946) estimator $\bar{z}^*$ for population III

N_1	N_2	l	$\bar{z}^*$	$\bar{z}^*_{CR}$	$\bar{z}^*_{RR}$	$\bar{z}^*_{SKR}$	T_k
4500	500	2	100.00	247.73	206.52	61.95	261.21
		4	100.00	262.96	173.01	71.83	259.98
		6	100.00	274.65	155.54	80.76	259.13
		8	100.00	283.91	144.82	88.85	258.51
4250	750	2	100.00	247.07	194.85	62.04	261.90
		4	100.00	262.98	159.09	75.60	260.12
		6	100.00	273.86	142.91	87.37	259.04
		8	100.00	281.77	133.69	97.69	258.31
4000	1000	2	100.00	237.97	188.89	63.45	258.44
		4	100.00	240.37	152.51	79.03	252.88
		6	100.00	241.78	137.26	92.07	249.78
		8	100.00	242.71	128.87	103.15	247.80

6. Interpretations and Conclusion of Empirical Results

The following interpretations may be carried out from Table 2-4:

a). From Tables 2-4, it may be observed that the proposed class of estimators T_k has impressive gains in efficiencies over the existing estimators for all different choice of l (inverse sampling ratio).

b). Further, it is to be noted that for all Population, the PRE of $\bar{z}^*_{SKR}$ increases respectively, with the increase of nonresponse rate and increase of l (inverse sampling ratio) while PRE of $\bar{z}^*_{RR}$ is decreasing with increase in inverse sampling ratio.

c). For Population I, II and III, it is shown that

i). The PRE of proposed class of estimator T_k decreases with the increases in the value of l (inverse sampling ratio) for all the three population except N1 = (4000) and N2 = (1000) for population I

ii). The PRE of the estimator $\bar{z}^*_{CR}$ decreasing and increasing respectively for population II and population III with increase in inverse sampling ratio except for N_1 = (4250) and N_2 = (750) for population II. For population I, it follows same trend as follows by T_k.

Also, we can see that the almost similar patterns are observed as described in above points for other remaining estimators.

Thus, from the interpretations of preceding analyses, it is clear that our proposed class of estimator for all three populations is more justifiable than other existing estimators of similar nature. The study also reveals that proposed estimator is more robust with the increase in inverse sampling ratio as compared to other estimators. Thus, our proposed class of estimators is more economical and can be recommended to study the characteristics of the variable in interest where measurement errors and non-response occur in the survey.

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REFERENCES

AZEEM, M. AND HANIF, M., 2017, Joint influence of measurement error and non-response on estimation of population mean. *Communications in Statistics-Theory and Methods*, 46(4), 1679-1693.

COCHRAN, W.G., 1968, Errors of measurement in statistics, *Technometrics*, 10, 637-666.

COCHRAN, W.G., 1977, *Sampling techniques*, 3rd edn., John Wiley & Sons, New York.

DIXON, J., 2010, Assessing nonresponse bias and measurement error using statistical matching in R. Harter, ed., *Section on Survey Research Methods - JSM*, American Statistical Association, 3388–3396.

FULLER, W., 1995, Estimation in the presence of measurement error, *International Statistical Review*, 63, 121–147.

HANSEN, M.H. AND HURWITZ, W.N., 1946, The problem of non-response in sample surveys. *Journal of American Statistical Association*, 41, 517–529.

JACKMAN, S., 1999, Correcting surveys for non-response and measurement error using auxiliary information, *Electoral Studies*, 18, 7–27.

MANEESHA AND SINGH, R.K., 2002, Role of regression estimator involving measurement errors, *Brazilian Journal of Probability and Statistics*, 16, 39-46.

OKAFOR, F.C. AND LEE, H., 2000, Double sampling for ratio and regression estimation with sub-sampling the nonrespondent, *Survey Methodology*, 26, 183–188.

RAO, P.S.R.S., 1986, Ratio estimation with subsampling the nonrespondents, *Survey Methodology*, 12, 217–230.

RAO, P.S.R.S., 1990, Regression estimators with subsampling of nonrespondents, In Gunar E. Liepins and V.R.R. Uppuluri, (eds) *In-Data Quality Control, Theory and Pragmatics*, Marcel Dekker, New York, 191-208.

SHALABH, 1997, Ratio method of estimation in the presence of measurement errors, *Journal of the Indian Society Agricultural Statistics*, 50, 150-155.

SINGH, H.P. AND KUMAR, S., 2008, Estimation of mean in presence of nonresponse using two-phase sampling scheme, *Statistical Papers*, 51, 559-582.

SINGH, N. AND VISHWAKARMA, G.K., 2019, A generalised class of estimator of population mean with the combined effect of measurement errors and non-response in sample survey, *Revista de Investigacion Operacional*, 40(2), 275-285.

SINGH, V.K. AND SHUKLA, D., 1987, One parameter family of factor-type ratio estimators, *Metron*, 45(1-2), 273-283.

SODIPO, A.A. AND OBISESAN, K.O., 2007, Estimation of the population mean using difference cum ratio estimator with full response on the auxiliary character, *Research Journal of Applied Sciences*, 2, 769-772.

SRIVASTAVA, A.K. AND SHALABH., 2001, Effect of measurement errors on the regression method of estimation in survey sampling, *Journal of Statistical Research*, 35, 35-44.

TABASUM, R. AND KHAN, I.A., 2004, Double sampling for ratio estimation with non-response, *Journal of the Indian Society of Agricultural Statistics*, 58, 300–306.

VISHWAKARMA, G.K. AND SINGH, R., 2016, Estimation of mean with nonresponse in double sampling using exponential technique, *Journal of Computations & Modelling*, 6, 1, 135- 149.