Recursive Quantile Estimation through a Stochastic Algorithm

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In this paper, we propose an estimate of a quantile of an unknown population. By considering this problem as a stochastic approximation problem, we obtain an estimator of the quantile and provide the almost-sure convergence as well as the asymptotic normality of this estimator. Some simulation results are presented to show that the proposed estimator works well.

Keywords: stochastic approximation, non-parametric estimation, quantile estimation

1. Introduction

The population mean of a variable X provides an important central measure, while the population median is an important alternative that is robust to potential outliers.

The quantiles, a generalized concept of median, are capable of providing not only central features but also the tail properties of the response distribution. Quantile plays a fundamental role in various statistical applications. They often arise as the natural parameters to estimate when the distribution is skewed.

Let $0 < q < 1$ be a probability. The q^{th} quantile is the smallest number $x \in \mathbb{R}$ so that $P(X > x) \leq 1 - q$, given a random variable X with a continuous distribution function F (.). For a given $q \in (0, 1)$, we address the estimation of the quantile of a random variable X defined in a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, with values on $\mathbb{R}$. Let X be a random variable and F its distribution function, the quantile function is defined by

$$\theta_q + F^{-1}(q) = inf\{x, F(x) \geq q\}$$